

Fig. 5. Scattered field in the vicinity of a soda-borosilicate glass slab by a parallel ($\phi = 180^\circ$) plane wave. The orientation is the same as shown in Fig. 1 and the slab cross section is indicated by the dark heavy line.

the center but shifted toward the back of the dielectric slabs in contrast to the normal incidence case where maxima occur at the point closest to the center of the slabs. The maxima of the scattered fields for parallel incidence are generally greater than those of normal incidence. This is in variance to some conjectures that when a Plexiglas wall lies parallel to the direction of propagation of a plane wave it creates only a slight field perturbation relative to that created by other wall orientations.

Keeping in mind the caution necessary in interpretation of the results that were mentioned in the Introduction, the results of this short paper suggest that for minimal perturbation the broad face of an animal holder should be oriented toward the direction of propagation. Moreover, the individual members of the restraining device should be as thin as practicable and the dielectric materials should be chosen to correspond as much as possible to the characteristics of free space. It is also clear that for the least amount of perturbation on both the incident and induced fields, it is necessary to place the animal in a region where the scattered fields due to restrainer alone are insignificant. This region is generally located in the forward portion of a restraining device. For an animal placed in the aft portion of the restraining device, the power absorbed could be altered significantly for parallel incidence because the field scattered by the isolated restrainer is strong in that region. However, the precise amount of perturbation will depend on the material used and the shape and size of the restraining device, as well as the animal involved, and is beyond the scope of this short paper. It should be emphasized that at the moment it is impossible to expose animals without some degree of restraining.

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Bidirectionality in Gyrotropic Waveguides

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Abstract—A waveguide containing gyrotropic media is shown to be bidirectional if it possesses at least one of the following symmetry operations: reflection in a plane perpendicular to the waveguide axis; 180° rotation about an axis perpendicular to the waveguide axis; or inversion in a point on the waveguide axis. The relation between the modal electromagnetic field components for any complementary mode pair is given for each symmetry case.

I. INTRODUCTION

The modes of a uniform waveguide have electromagnetic fields which vary longitudinally as $\exp(-\gamma_n z)$, where γ_n is the propagation constant for the n th mode and z is the waveguide axis. For a fixed frequency the mode spectrum of a uniform waveguide is infinite. If the waveguide's transverse boundaries are closed (for example, perfect conductors), there is an infinite discrete spectrum. For all uniform waveguides of current technical interest whose transverse boundaries are open (the fields extend to infinity), there are a finite number of discrete modes plus a continuous spectrum. A uniform waveguide is termed bidirectional if, at any frequency, for each mode with propagation constant γ_n there exists a complementary mode with propagation constant $-\gamma_n$.

It is well known that if the media filling a uniform waveguide have permeability and permittivity dyadics which are symmetric ($\tilde{\mu} = \mu$, $\tilde{\epsilon} = \epsilon$), the waveguide will be both bidirectional and reciprocal. However, the absence of media with nonsymmetric permeability and permittivity dyadics is only a sufficient and not a necessary condition for bidirectionality. Under some conditions waveguides containing media with nonsymmetric permeability and/or permittivity dyadics, such as gyrotropic media, may be bidirectional. This short paper establishes sufficient conditions for bidirectionality in uniform waveguides containing gyrotropic media. The discussion applies to inhomogeneous, lossy, uniform waveguides with open or closed boundaries.

Let S be a generalized medium susceptibility which relates a

response of the medium to a driving force; for example, the electric susceptibility relates the electric polarization to the applied electric field. If the relationship is linear, the Onsager relations [1], which are based on the principle of microscopic reversibility, state that the elements of the susceptibility dyadic must satisfy: $S_{ij}(\mathbf{H}, \mathbf{v}) = S_{ji}(-\mathbf{H}, -\mathbf{v})$. Here the possibility that the generalized susceptibility depends on an applied magnetic field \mathbf{H} , or on a macroscopic drift velocity \mathbf{v} of the constituent particles of the medium, is included. In this short paper only media with no macroscopic drift of the constituent particles are considered ($\mathbf{v} = 0$). Therefore we can set $S_{ij}(\mathbf{H}) = S_{ji}(-\mathbf{H})$. For such media the Onsager relations state that the susceptibilities must be symmetric dyadics ($S_{ij} = S_{ji}$) unless the particular susceptibility is a function of an external magnetic field, and such a magnetic field is applied to the medium. If an applied magnetic field produces a nonsymmetric susceptibility dyadic, the medium is termed gyrotropic. Since a medium (with the restriction $\mathbf{v} = 0$) which is nongyrotropic must have symmetrical susceptibility dyadics, any uniform waveguide containing only such media must be both bidirectional and reciprocal. Considering only media with $\mathbf{v} = 0$ is not unduly restrictive; most passive dielectric and magnetic media of current technological interest meet this condition.

The most common gyrotropic media are plasmas and ferrites. For a plasma with the biasing magnetic field in the z direction, the permittivity dyadic (linearized approximation) is [2]

$$\epsilon = \begin{pmatrix} \epsilon_1 & \epsilon_2 & 0 \\ -\epsilon_2 & \epsilon_1 & 0 \\ 0 & 0 & \epsilon_3 \end{pmatrix}. \quad (1)$$

For a ferrite with the biasing magnetic field in the z direction, the permeability dyadic is [3]

$$\mu = \begin{pmatrix} \mu & -j\kappa & 0 \\ j\kappa & \mu & 0 \\ 0 & 0 & \mu_0 \end{pmatrix}. \quad (2)$$

In general, the element values of these dyadics are functions of the frequency and the biasing magnetic field strength and are complex. Reversing the direction of the biasing magnetic field reverses the sign of ϵ_2 in (1) and of κ in (2), but not of the other elements in these dyadics. Therefore ϵ and μ satisfy the previously given Onsager relations.

The conditions for bidirectionality in waveguides containing gyrotropic media are found to be related to the possession of certain symmetries by the waveguide. The discussion will be expressed in terms of physical arguments, although it could be cast in a more mathematical form using group theoretic concepts.

II. CONDITIONS FOR BIDIRECTIONALITY

It is obvious that if two waveguides are identical, then their mode spectra must also be identical. This fact will be exploited in the following discussion. Consider a waveguide for which a spatial symmetry operation exists that interchanges the $+z$ and $-z$ axes of the waveguide. By definition of a symmetry operation, the transformed waveguide must appear identical to the original structure. Specific examples of possible symmetry operations will be discussed later. Since the transformed waveguide is identical to the original waveguide, they must have identical mode spectra. That is, for each mode with propagation constant γ_n of the original waveguide, there is a corresponding mode with propagation constant γ_n in the transformed waveguide. Because the original and transformed waveguides differ by an interchange of the $+z$

and $-z$ axes, looking in the $+z$ direction in the transformed waveguide is equivalent to looking in the $-z$ direction in the original waveguide. Thus a mode with propagation constant γ_n in the transformed waveguide is equivalent to a mode with propagation constant $-\gamma_n$ in the original waveguide. We conclude that a waveguide with this spatial symmetry must have complementary pairs of modes with $+\gamma_n$ and $-\gamma_n$ propagation constants and is, therefore, bidirectional.

There are just three spatial symmetry operations that interchange the $+z$ and $-z$ axes. These operations are as follows: 1) reflection in a plane perpendicular to the z axis; 2) rotation by 180° about an axis perpendicular to the z axis; and 3) inversion in a point on the z axis. Any uniform waveguide possessing one or more of these symmetry operations must be bidirectional. Of course, this is a sufficient, not a necessary, condition for bidirectionality, because waveguides containing only media with symmetric permeability and permittivity dyadics are bidirectional regardless of whether any of these symmetries are present. Two examples of these latter waveguides are as follows: 1) waveguides with an irregular (asymmetric) conductor cross section containing biaxial dielectric media, none of whose principal axes is parallel to the waveguide axis, and 2) waveguides containing anisotropically conducting sheets so oriented that none of the three symmetry operations is present (for example, a sheath helix inside a rectangular waveguide with the helix axis parallel to the z axis but located on neither of the symmetry planes of the rectangular waveguide).

The invariance of the waveguide after a symmetry operation is performed applies to both the geometric form of the waveguide structure and to the permeability and permittivity dyadics of the gyrotropic media in the waveguide. An essential feature of the gyrotropic medium is the dc magnetic field H_0 which biases the medium and determines the element values of the permeability and/or permittivity dyadics at a given frequency. Thus the symmetry properties of the gyrotropic medium include the symmetry properties of this biasing magnetic field. Since a magnetic field is an axial vector (pseudovector), its transformation properties under reflection or inversion are different from those of an electric field, which is a polar vector (true vector). The transformation properties of electric and magnetic fields under reflection, rotation, and inversion are summarized in the Appendix.

The results presented in the discussion to follow for waveguides with gyrotropic media which are invariant to reflection in a plane perpendicular to the z axis are not new [4]. They are repeated here for completeness. However, the results for waveguides containing gyrotropic media which possess either of the other two symmetry operations to be discussed have not been previously published.

Suhl and Walker [5] mention a connection between "geometrical symmetry" and reciprocity in waveguides with a purely transverse biasing magnetic field. They do not explore this connection to determine the particular symmetry required, nor the role played by the pseudovector character of the biasing magnetic field. In their example of a reciprocal waveguide (a rectangular waveguide with a centered ferrite slab), they ascribe reciprocity to the presence of a transverse plane of reflection symmetry; in fact, there is no such symmetry plane (because H_0 is a pseudovector), and bidirectionality occurs in this case because of the presence of the second and third symmetry conditions to be discussed. In a qualitative discussion of the H_{n0} modes in a rectangular waveguide with transversely magnetized ferrite slabs, Kales [6] also mentions a connection between symmetry and reciprocity. He does not explore the particular

symmetry required, nor attempt to formulate the presumed connection precisely. In both of these papers, symmetry and reciprocity are coupled. Actually, symmetry determines bidirectionality; the presence of reciprocal behavior is a more complex question (this is discussed briefly in Section III).

First, we will examine waveguides containing gyrotropic media which are invariant to a reflection in a plane perpendicular to the z axis. From the transformation properties of a magnetic field under reflection [see (8) in the Appendix], it is clear that reflection in a plane perpendicular to the z axis is a symmetry operation for such a waveguide only if the biasing magnetic field H_0 is parallel to the z axis. We can conclude, therefore, that any waveguide containing gyrotropic media is bidirectional if the waveguide structure is geometrically invariant to reflection in a plane perpendicular to the z axis, and the biasing magnetic field is parallel to the z axis. Two examples of waveguides in this category are shown in Fig. 1.

It is also possible to relate the components of the electric and magnetic fields of two complementary modes with propagation constants γ_n and $-\gamma_n$. The electromagnetic field of the n th mode is

$$\begin{aligned} E_n(x, y, z) &= \{E_{Tn}(x, y) + \mathbf{a}_z E_{zn}(x, y)\} \exp[-\gamma_n z] \\ H_n(x, y, z) &= \{H_{Tn}(x, y) + \mathbf{a}_z H_{zn}(x, y)\} \exp[-\gamma_n z] \end{aligned} \quad (3)$$

where \mathbf{a}_z is a unit vector in the z direction, and the subscript T indicates a transverse component. If the symmetry operation of reflection in the $z = 0$ plane is applied to the waveguide but not to the electromagnetic field, then the modal electromagnetic field of (3) is a solution for the transformed waveguide. Conversely, if the symmetry operation of reflection in the $z = 0$ plane is applied to the modal electromagnetic field of (3) but not to the waveguide structure, then the transformed electromagnetic field is a solution for the original waveguide. Using the relations given in (8) of the Appendix, the transformed electromagnetic field after reflection in the $z = 0$ plane is

$$\begin{aligned} E_n'(x, y, z) &= \{E_{Tn}(x, y) - \mathbf{a}_z E_{zn}(x, y)\} \exp[\gamma_n z] \\ H_n'(x, y, z) &= \{-H_{Tn}(x, y) + \mathbf{a}_z H_{zn}(x, y)\} \exp[\gamma_n z]. \end{aligned} \quad (4)$$

However, this is the modal electromagnetic field for the complementary mode to the original model. Thus the electromagnetic field components for any complementary mode pair ($\gamma_n, \gamma_n' = -\gamma_n$) in a bidirectional waveguide with reflection symmetry are related as

$$\begin{aligned} E_{Tn}'(x, y) &= E_{Tn}(x, y) & H_{Tn}'(x, y) &= -H_{Tn}(x, y) \\ E_{zn}'(x, y) &= -E_{zn}(x, y) & H_{zn}'(x, y) &= H_{zn}(x, y). \end{aligned} \quad (5)$$

Next we will examine waveguides containing gyrotropic media which are invariant to a rotation of 180° about an axis perpendicular to the z axis. Without loss of generality the axis of rotation symmetry is chosen to be the x axis. From the transformation properties of a magnetic field under rotation [see (9) in the Appendix], it is clear that rotation by 180° about the x axis is a symmetry operation for a waveguide containing gyrotropic media

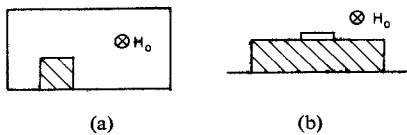


Fig. 1. Waveguides with reflection symmetry (longitudinally magnetized ferrite slabs). (a) Rectangular waveguide. (b) Microstrip.

only if the biasing magnetic field H_0 is parallel to the x axis. We can conclude, therefore, that any such waveguide is bidirectional if the waveguide structure is geometrically invariant to 180° rotation about an axis perpendicular to the z axis, and the biasing magnetic field is parallel to the axis of rotation symmetry. Two examples of waveguides in this category are shown in Fig. 2. Consideration of the transformation relations for electric and magnetic fields under 180° rotation given in (9) of the Appendix shows that the electromagnetic field components for any complementary mode pair ($\gamma_n, \gamma_n' = -\gamma_n$) in a bidirectional waveguide with 180° rotation symmetry are related as

$$\begin{aligned} E_{xn}'(x, y) &= E_{xn}(x, -y) & H_{xn}'(x, y) &= H_{xn}(x, -y) \\ E_{yn}'(x, y) &= -E_{yn}(x, -y) & H_{yn}'(x, y) &= -H_{yn}(x, -y) \\ E_{zn}'(x, y) &= -E_{zn}(x, -y) & H_{zn}'(x, y) &= -H_{zn}(x, -y). \end{aligned} \quad (6)$$

Finally, we will examine waveguides containing gyrotropic media which are invariant to an inversion in any point on the z axis (specifically, the $z = 0$ point will be considered). From the transformation properties of a magnetic field under inversion [see (10) of the Appendix], it is clear that inversion is a symmetry operation for a waveguide containing gyrotropic media for any direction of the biasing magnetic field. We can conclude, therefore, that any such waveguide is bidirectional if the waveguide structure is geometrically invariant to inversion in a point on the waveguide axis, regardless of the direction of the biasing magnetic field. Consideration of the transformation relations for electric and magnetic fields under inversion given in (10) of the Appendix shows that the electromagnetic field components for any complementary mode pair ($\gamma_n, \gamma_n' = -\gamma_n$) in a bidirectional waveguide with inversion symmetry are related as

$$E_n'(x, y) = -E_n(-x, -y) \quad H_n'(x, y) = H_n(-x, -y). \quad (7)$$

Two examples of waveguides in this category are shown in Fig. 3.

A waveguide containing gyrotropic media may simultaneously possess two of these symmetries: either the first and third, or the second and third (the first and second are incompatible). Such waveguides will also be bidirectional, of course. If two of these symmetries are simultaneously present, further conclusions concerning the transverse variation of the electromagnetic fields can be drawn. For example, if the waveguide possesses both reflection and inversion symmetry [see Fig. 4(a) for an example],

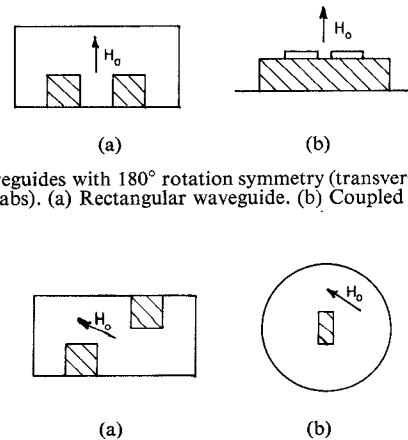


Fig. 2. Waveguides with 180° rotation symmetry (transversely magnetized ferrite slabs). (a) Rectangular waveguide. (b) Coupled microstrip.

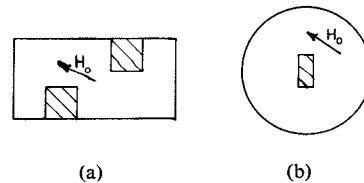


Fig. 3. Waveguides with inversion symmetry (ferrite slabs with arbitrary magnetization direction). (a) Rectangular waveguide. (b) Circular waveguide.

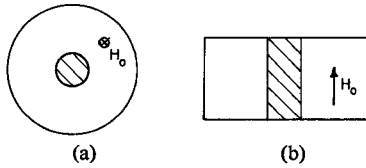


Fig. 4. Waveguides with combined symmetries. (a) Circular waveguide with axially magnetized ferrite rod (reflection and inversion symmetries). (b) Rectangular waveguide with transversely magnetized ferrite slab (180° rotation and inversion symmetries).

then the modal electromagnetic field components must be either even functions of both x and y or odd functions of both x and y . If the waveguide possesses both 180° rotation and inversion symmetry [see Fig. 4(b) for an example], then the modal electromagnetic field components must be either even or odd functions of x (the 180° rotation axis); there are no restrictions on the y variation.

III. CONCLUSIONS

It has been shown that a waveguide containing gyrotropic media with a biasing magnetic field H_0 will be bidirectional if it possesses one of the following symmetries: reflection symmetry in a plane perpendicular to the waveguide axis with H_0 parallel to the waveguide axis; 180° rotation symmetry about an axis perpendicular to the waveguide axis with H_0 parallel to the rotation axis; or inversion in a point on the waveguide axis with no restriction on the direction of H_0 . It is conjectured that the possession of one of these symmetries is also a necessary condition for a waveguide containing gyrotropic media to be bidirectional, but this has not been proved.

In the examples shown in Figs. 1–4, the biasing magnetic field is uniform in direction and magnitude over the waveguide cross section. The relationship between bidirectionality and the symmetry conditions holds more generally for cases where the direction or magnitude of the biasing magnetic field varies over the cross section. However, any such variation must be properly accounted for in the determination of the symmetry properties of the gyrotropic media in the waveguide.

A comment concerning the relation between reciprocity and bidirectionality is warranted. An arbitrary current excitation at plane z_1 of a waveguide will produce a related electromagnetic field at plane z_2 . If this same current excitation, when located at plane z_2 , produces an identical electromagnetic field at plane z_1 , then the waveguide is said to be reciprocal. A bidirectional waveguide containing gyrotropic media will, in general, not exhibit this reciprocity. For example, suppose a linearly polarized current excites the dominant modes of the bidirectional waveguide shown in Fig. 4(a) (the dominant modes are related to the H_{11} modes of an empty circular waveguide). This excitation will produce an electromagnetic field which exhibits Faraday rotation [7], which is a nonreciprocal effect. On the other hand, if the current excitation is such as to excite only those modes of this waveguide which are related to the E_{0n} modes of an empty circular waveguide, reciprocal behavior will be observed [8]. The symmetry conditions establish whether a waveguide containing gyrotropic media will be bidirectional, but they do not determine whether or not reciprocal behavior might be observed for some particular excitation.

APPENDIX

The transformation properties of electric fields (polar vectors) and magnetic fields (axial vectors) under reflection, 180° rotation, and inversion are summarized here. For a more complete dis-

cussion of the properties of polar and axial vectors see [9] or [10]; [11] presents an excellent comparison of the transformation properties of electric and magnetic fields based on physical arguments. Using primes to label the transformed field components, reflection of electric and magnetic fields in the $z = 0$ plane yields

$$\begin{aligned} E_T'(x, y, z) &= E_T(x, y, -z) & H_T'(x, y, z) &= -H_T(x, y, -z) \\ E_z'(x, y, z) &= -E_z(x, y, -z) & H_z'(x, y, z) &= H_z(x, y, -z). \end{aligned} \quad (8)$$

Rotation of electric and magnetic fields by 180° about the x axis yields

$$\begin{aligned} E_x'(x, y, z) &= E_x(x, -y, -z) & H_x'(x, y, z) &= H_x(x, -y, -z) \\ E_y'(x, y, z) &= -E_y(x, -y, -z) & H_y'(x, y, z) &= -H_y(x, -y, -z) \\ E_z'(x, y, z) &= -E_z(x, -y, -z) & H_z'(x, y, z) &= -H_z(x, -y, -z). \end{aligned} \quad (9)$$

Inversion of electric and magnetic fields in $z = 0$ yields

$$E'(x, y, z) = -E(-x, -y, -z) \quad H'(x, y, z) = H(-x, -y, -z). \quad (10)$$

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The Piezoelectric-Magnetoelastic Wave Propagation Through the Conducting Plate in a Composite Medium

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Abstract—The piezoelectric and magnetoelastic surface wave propagation through a composite layered structure of one piezoelectric and another magnetoelastic media is considered with a metal plate placed in between them. The dispersion relations have been derived and numerically computed. Thereafter, the field distributions are evaluated.

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